

The idea of a classic mathematical proof is to begin with a series of axioms, statements that can be assumed to be true or that are self-evidently true. Then by arguing logically, step by step, it is possible to arrive at a conclusion. If the axioms are correct and the logic is flawless, then the conclusion will be undeniable. This conclusion is the theorem.

Mathematical theorems rely on this logical process and once proven are true until the end of time. Mathematical proofs are absolute. To appreciate the value of such proofs they should be compared with their poor relation, the scientific proof. In science a hypothesis is put forward to explain a physical phenomenon. If observations of the phenomenon compare well with the hypothesis, this becomes evidence in favor of it. Furthermore, the hypothesis should not merely describe a known phenomenon, but predict the results of other phenomena. Experiments may be performed to test the predictive power of the hypothesis, and if it continues to be successful then this is even more evidence to back the hypothesis. Eventually the amount of evidence may be overwhelming and the hypothesis becomes accepted as a scientific theory.

However, the scientific theory can never be proved to the same absolute level of a mathematical theorem. It is merely considered highly likely based on the evidence available. So-called scientific proof relies on observation and perception, both of which are fallible and provide only approximations to the truth. As Bertrand Russell pointed out: "Although this may seem a paradox, all exact science is dominated by the idea of approximation." Even the most widely accepted scientific "proofs" always have a small element of doubt in them. Sometimes this doubt diminishes, although it never disappears completely, while on other occasions the proof is ultimately shown to be wrong. The weakness in scientific proof leads to scientific revolutions in which one theory that was assumed to be correct is replaced with another theory, which may be merely a refinement of the original theory, or which may be a complete contradiction.

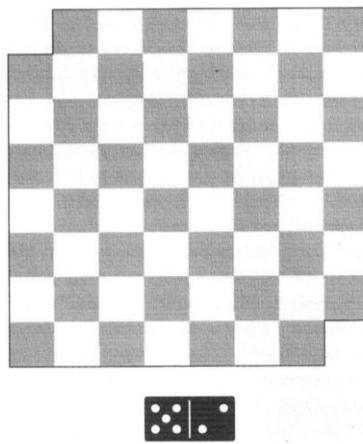
For example, the search for the fundamental particles of matter involved each generation of physicists overturning or, at the very least, refining the theory of their predecessors. The modern quest for the building blocks of the universe started at the beginning of the nineteenth century when a series of experiments led John Dalton to suggest that everything was composed of discrete atoms, and that atoms were fundamental. At the end of the century J.J. Thomson discovered the electron, the first known subatomic particle, and therefore the atom was no longer fundamental. During the early years of the twentieth century, physicists developed a "complete" picture of the atom—a nucleus consisting of protons and neutrons, orbited by electrons. Then cosmic ray experiments revealed the existence of other fundamental particles—pions and muons. An even greater revolution came with the discovery in 1932 of antimatter—the existence of antiprotons, antineutrons, antielectrons, etc. By the time particle physicists could not be sure how many different particles existed, but at least they could be confident that these entities were indeed fundamental. That was until the 1960's when the concept of quark was born. The proton itself is apparently built from fractionally charged quarks, as is the neutron and the pion. In the next decade the very concept of a particle as a pointlike object may even be replaced by the idea of particles as strings. The theory is that strings a billionth of a billionth of a billionth of a meter in length (so small that they appear pointlike) can vibrate in different ways, and each vibration gives rise to a different particle. This is analogous to Pythagoras's discovery that one string on a lyre can give rise to different notes depending on how it vibrates. The moral

of the story is that physicists are continually altering their picture of the universe, if not rubbing it out and starting all over again.

The science fiction writer and futurologist Arthur C. Clarke wrote that if an eminent professor states that something is undoubtedly true, then it is likely to be proved false the next day. Scientific proof is inevitably fickle and shoddy. On the other hand, mathematical proof is absolute and devoid of doubt. Pythagoras died confident in the knowledge that his theorem, which was true in 500 a.c. , would remain true for eternity.

Science is operated according to the judicial system. A theory is assumed to be true if there is enough evidence to prove it “beyond all reasonable doubt”. On the other hand, mathematics does not rely on evidence from fallible experimentation, but it is built on infallible logic. This is demonstrated by the problem of the “mutilated chessboard”, illustrated in *Figure-1*.

Figure-1



We have a chessboard with two opposing corners removed, so that there are only 62 squares remaining. Now we take 31 dominoes shaped such that each domino covers exactly two squares. The question is: It is possible to arrange the 31 dominoes so that they cover all the 62 squares on the chessboard?

There is two approaches to this problem:

(1) The scientific approach

The scientific would try to solve the problem by experimenting, and after trying out a few dozen arrangements would discover that they all fail. Eventually, the scientist believes that there is enough evidence to say that the board cannot be covered. However, the scientist can never be sure that this is truly the case, because there might be some arrangement that has not been tried that might do the trick. There are millions of different arrangements, and it is possible to explore only a small fraction of them. The conclusion that the task is impossible is a theory based on experiment, but the scientist will have to live with the prospect that one day the theory may be overturned.

(2) The mathematical approach

The mathematician tries to answer the question by developing a logical argument that will derive a conclusion that is undoubtedly correct and that will remain unchallenged forever. One such argument is the following:

- The corners that were removed from the chessboard were both white. Therefore there are now 32 black squares and only 30 white squares.
- Each domino covers two neighboring squares, and neighboring squares are always different in color, i.e., one black and one white.
- Therefore, no matter how they are arranged, the first 30 dominoes laid on the board must cover 30 white squares, and 30 black squares.
- Consequently, this will always leave you with one domino and two black squares remaining.
- But remember, all dominoes cover two neighboring squares, and neighboring squares are opposite in color. However, the two squares remaining are the same color and so they cannot both be covered by the one remaining domino. Therefore, covering the board is impossible!

This proof shows that every possible arrangement of dominoes will fail to cover the mutilated chessboard.

From:

Fermat's Enigma, by Simon Singh

The Saga of Mathematics by Lewinter-Widulski