

Game Theory

Game theory is a branch of applied mathematics that came into being with the 1944 book Theory of Games and Economic Behavior by John von Neumann and Oskar Morgenstern. It is used in the social sciences, most notably in economics, as well as in biology, engineering, political science, international relations, computer science, and philosophy. Game theory attempts to mathematically capture behavior in strategic situations, in which an individual's success in making choices depends on the choices of others.

In its simplest form, two players are playing the same game. The game can be *zero-sum* (what a player wins, the other one loses) or *non-zero-sum* (both can win, or lose).

A. The prisoner's dilemma

Two suspects are arrested by the police. The police have insufficient evidence for a conviction, and, having separated both prisoners, visit each of them to offer the same deal. If one testifies against the other and the other remains silent, the betrayer goes free and the silent accomplice receives the full 10-year sentence. If both remain silent, both prisoners are sentenced to only six months in jail for a minor charge. If each betrays the other, each receives a five-year sentence. Each prisoner must choose to betray the other or to remain silent. How should the prisoners act?

The game is usually represented by a matrix which shows the players, strategies, and payoffs.

	Prisoner 1 Stays Silent	Prisoner 1 Betrays
Prisoner 2 Stays Silent	-1/2 -1/2	-10 0
Prisoner 2 Betrays	0 -10	-5 -5

What strategy would you choose?

B. Harry's dilemma

Harry is wondering whether he should revise the lesson for his next English class, just in case there is a surprise test.

I. Who wins what ?

1. Identify the two players.
What can each of them possibly win ?
2. Draw a “payoff matrix” for this game.

II. Let’s try

1. a) If Harry decides to learn his lesson, what will the worst situation be ?
b) If he decides not to learn his lesson, what will the worst situation be ?
c) The best solution for Harry is to choose the “least-worst” possibility (the “minimax” strategy).
So, is he going to learn his lesson or not ?
2. And what about the teacher ?

III. Can Harry do better?

Surely my teacher will not give us a surprise test each lesson! thinks Harry. *What about learning my lesson just a bit less often?*

Suppose Harry decides to revise 70 % of time and that his teacher decides to give the class a test 40 % of time.

1. a) What percentage of time will Harry not revise?
b) On average, what will his payoff be if there is a test?
On average, what will his payoff be if there isn’t a test?
c) Fill in the payoff columns for Harry (below).
d) If he sticks to the minimax strategy, what is the “least-worst” of those three strategies (R , \bar{R} or M_{70}) ?
2. And what about the teacher’s point of view?

Harry:

	R	M_{70}	\bar{R}
T			
\bar{T}			

Teacher:

	R	\bar{R}
T		
M_{40}		
\bar{T}		