Napier's Logarithms

I. A few formulae

A. Do you remember these?

$$10^n \times 10^m = \qquad\qquad ; \qquad \frac{10^n}{10^m} =$$

$$10^{-n} = \qquad\qquad ; \qquad (10^n)^m =$$

B. Have you heard of these?

1. a) Square $10^{0.5}$ to find out what that number could be.
b) Give two squares surrounding 10.
Why does $10^{0.5}$ stand between 3.1 and 3.2 ?
By trial and error, find an approximation of $10^{0.5}$ to 2 dp.

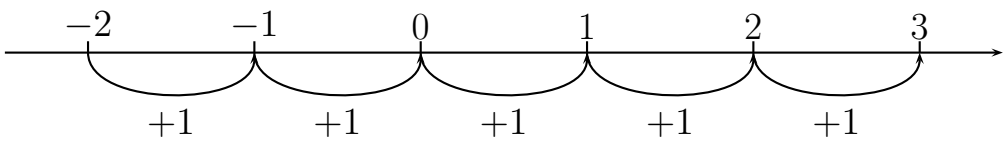
2. Can you find out what the number $10^{\frac{1}{3}}$ could be?
Again, by trial and error, find an approximation of $10^{\frac{1}{3}}$ to 2 dp.

3. Same question with $10^{0.25}$, $10^{2.5}$ and $10^{-1.5}$.

II. Arithmetic vs Geometric

A. Arithmetic progression

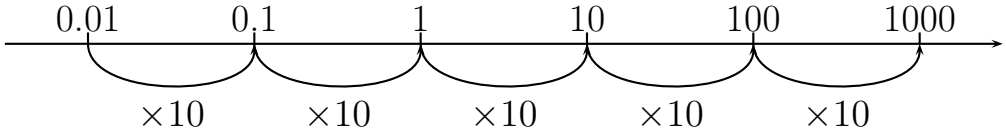
The marks on a "traditionnal" number line follow an *arithmetic progression.*
It is a linear scale.



B. Geometric progression

The marks on the "new" number line follow a *geometric progression.*
It is a nonlinear scale.

Powers of 10:



1. Where should the numbers of section **I.B.** go on this number line?

2. Where should 5 go on this number line? And 50?

III. Base 10 logarithms

John Napier (1550–1617)

A. Definition

If $a = 10^b$ then b is called the *logarithm* of a :

$$\boxed{\begin{array}{c} a = 10^b \iff b = \log(a). \\ \log(10^b) = b. \end{array}}$$

1. Use this definition to find the exact value of:

- $\log(100)$
- $\log(10^{-1})$
- $\log(10^{13})$
- $\log(10)$
- $\log(0.0001)$

2. Use your calculator to find an approximate value of:

- $\log(50)$
- $\log(5{,}000)$
- $\log(0.005)$

B. Rules of log

- $\log(10) = 1$
- $\log(a \times b) = \log(a) + \log(b)$
- $\log\left(\dfrac{a}{b}\right) = \log(a) - \log(b)$
- $\log(10^n) = n$
- $\log(a^n) = n\log(a)$
- $\log(a) = \log(b) \iff a = b$

1. Use these rules to write the following numbers in a more simple form:

a) $\log(1{,}000{,}000)$

b) $\log(2{,}000)$

c) $\log(0.0005)$

d) $\log\left(\dfrac{1}{25}\right)$

2. Use these rules to solve the following equations:

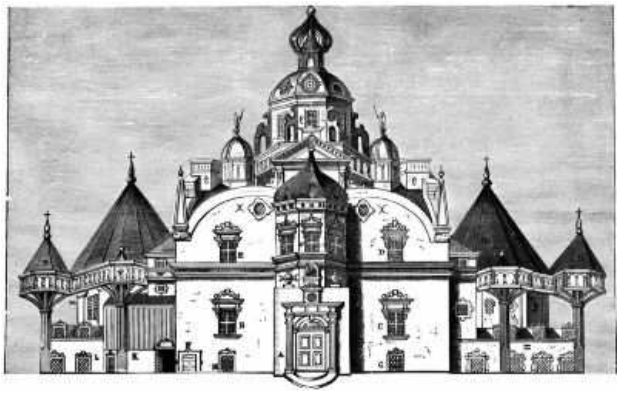
a) $\log(x) = 5 + \log(3)$

b) $\log(x^2) = 4$

IV. The third law of planetary motion

Logarithms were first used to help scientists perform long calculations.
One of them was Johannes Kepler. His academic mentor chided him for taking an interest in logarithms, remarking that "it is not seemly for a professor of mathematics to be childishly pleased about any shortening of the calculations"[1]. But Kepler had access to a huge quantity of data collected by the astronomer Tycho Brahe and wanted to use this data to model the motion of the planets.

Tycho Brahe (1546–1601)



Uranienborg, the astronomical observatory operated by Tycho Brahe, built circa 1576-1580 on the island of Hven (then in Denmark).

These are the values computed by Kepler:

	Average distance d (astronomical units)	Period p (years)
Mercury	0.39	0.24
Venus	0.72	0.62
Earth	1	1
Mars	1.52	1.88
Jupiter	5.20	11.86
Saturn	9.54	29.46

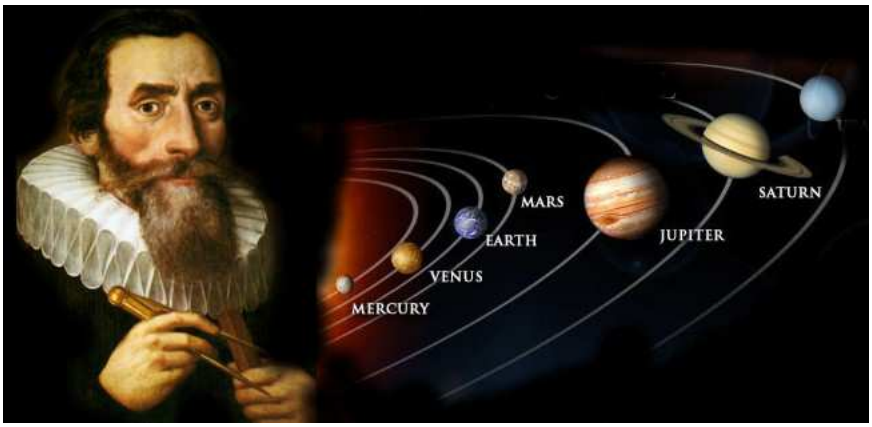
1. Plot the points corresponding to these values in a "traditionnal" graph with linear scales on each axis.
Why is this graph not very practical?

2. Work out the logarithms of each of the numbers in the previous table and plot $y = \log(p)$ in terms of $x = \log(d)$.

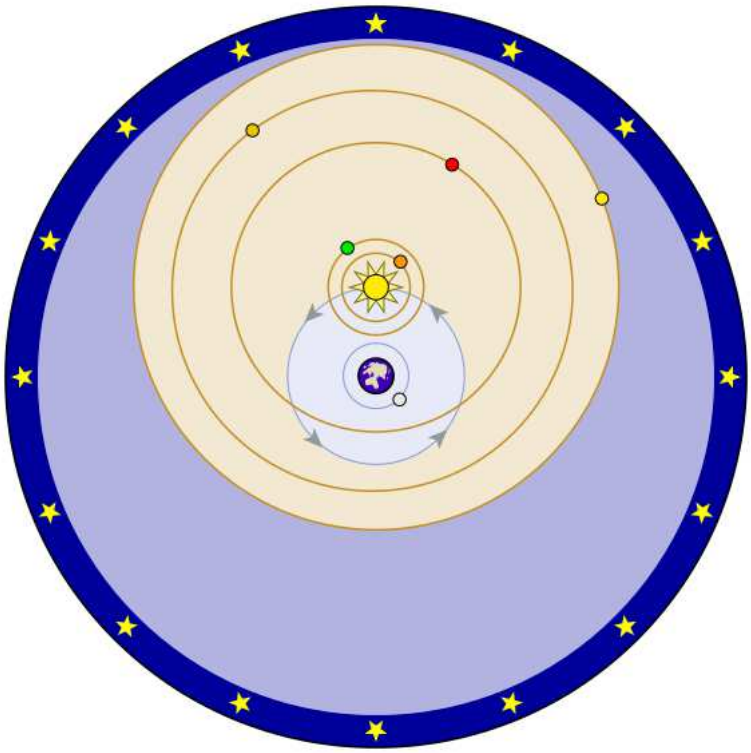
3. What kind of relationship can you conjecture between x and y?

[1] From `http://www.mathpages.com/rr/s8-01/8-01.htm`

4. Deduce from that a link between d and p.



Johannes Kepler (1571–1630)



The Tychonian system