

I. Non-transitive dice

A game¹ involves two players, each rolling a die, and the one with the higher number wins.

Each player starts by choosing a die from a selection of three, labelled A, B and C. Each die has three numbers, each one repeated to cover the six faces.

Die A – 3, 3, 5, 5, 7, 7

Die B – 2, 2, 4, 4, 9, 9

Die C – 1, 1, 6, 6, 8, 8

1. What happens if you play die A against die B: on average, how often does A beat B?
2. Similarly, analyse die B played against die C.
3. What do you think will happen if you play A against C?
4. Another paragraph of this article refers to the famous game of RPS, in which the two players show their hands in one of three ways : rock (clenched fist), paper (open, flat hand), or scissors (forefinger and middle finger form a 'V'). Explain how this game is related to non-transitive dice.
5. Simon Singh's article gives yet another interesting example:

Even better, Allen J Schwenk of Western Michigan University discovered a set of three non-transitive dice that exhibit a very peculiar (and useful) property. The dice have the following faces :

Die A – 1, 1, 1, 13, 13, 13

Die B – 0, 3, 3, 12, 12, 12

Die C – 2, 2, 2, 11, 11, 14

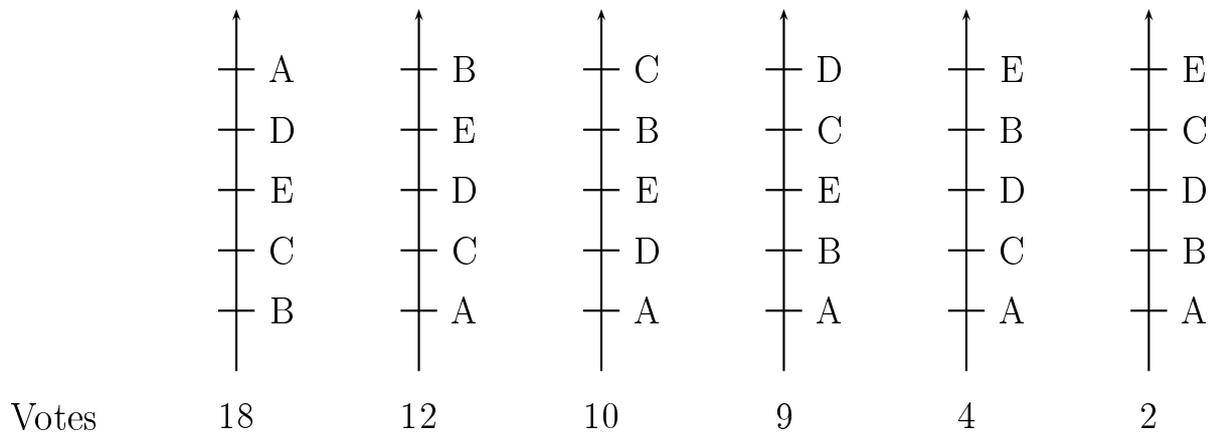
First, as before, if your opponent picks any die, then you can always pick one that beats it. However, if you are forced to pick first and your opponent happens to then pick the better die, then a slight rule change still gives you the edge. Just play the game such that each die is rolled twice and it is the highest total that wins. Bizarrely, your inferior die suddenly becomes superior.

Compute the probabilities for the second set of dice, when throwing the die twice, and check the last sentence in the document.

¹Described by Simon Singh in How To Pick A Winning Hand Every Time, guardian.co.uk

II. Election Decision Methods

Consider the election below²:



For example, 18 voters prefer A to D to E to C to B.

Here are five different procedures for selecting a winner for the election shown.

Plurality: count how many first place votes each candidate receives. The winner is the candidate with the largest number of first place votes.

Run-off election: count how many first place votes each candidate receives. If no candidate receives a majority, declare all candidates except those two who have gotten the largest number of first place votes as losers. Now, conduct a new election based on the preferences of the voters for these top two vote getters at this stage.

Sequential run-off election: if no candidate gets a majority based on first place votes, eliminate the candidate with the fewest first place votes and hold a new election based on voting only for the smaller collection of candidates. Repeat the process until some candidate receives a majority of the first place votes.

Borda count: given a preferential ballot and a candidate on the ballot, assign candidate X a number of points equal to the number of candidates below candidate X on the preference ballot. The Borda count procedure assigns as the winner of an election the candidate with the highest Borda count.

Condorcet: consider all possible two-way races between candidates. The Condorcet winner, if there is one, is the one candidate who can beat each other candidate in a two-way race with that candidate.

What do you notice if you carry out these 5 election methods on this 55 voter election?

²From <http://www.ams.org/samplings/feature-column/fcarc-voting-decision>